Metric Versus Ashtekar Variables in Two Killing Field Reduced Gravity

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Abstract

The relation between the $SL(2,\mathbb{R})/SO(2)$- and the $SL(2,\mathbb{C})$-chiral model that naturally arise within the metric respectively the Ashtekar formulation of two Killing field reduced Einstein gravity is revealed. Both chiral models turn out to be completely equivalent even though the transition from the coset- to the $SL(2,\mathbb{C})$-model is accompanied by a disappearance of the non-ultralocal terms in the Poisson brackets.

Among the various toy models which have been investigated in order to discuss in a simplified context some characteristic features of the canonical approach to quantum gravity, the two Killing field reduction of general relativity plays a particularly important rôle. Indeed, it belongs to the simplest reductions of full $4D$ general relativity that still describe nonlinear genuine field theories without a background metric. In this letter we will compare the outcomes of two different formulations of this reduction:

In the conventional metric picture, the symmetry reduction naturally leads to a $2D$ dilaton-gravity coupled $SL(2,\mathbb{R})/SO(2)$-σ-model. The underlying Lagrangian induces Poisson brackets with non-ultralocal terms (i.e. terms containing derivatives of δ-functions) between the corresponding σ-model currents.

On the other hand, performing the symmetry reduction within the Ashtekar formulation [1], naturally results in a description in terms of a generalized $SL(2,\mathbb{C})$-chiral model with completely ultralocal Poisson brackets [2, 3]. As for the equations of motion, however, one encounters a striking similarity to the $SL(2,\mathbb{R})/SO(2)$-formulation.

To understand the surprising coexistence of these two chiral models we will translate them into each other, verify whether they are equivalent and

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observe the mechanism that causes the differences between them (while preserving the similarities). The result will be interesting in its own right, but should also shed some light on the relation of the non-local charges constructed in [4] and [2].

To begin with, let us briefly recapitulate the symmetry reduction using the ordinary metric formalism:

A spacetime possessing two commuting spacelike and two-surface orthogonal Killing vector fields admits the choice of a local coordinate system (t, x, y, z) such that the Killing vectors are given by $\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z}$ and the metric G_{MN} attains the following form

$$G_{MN} = G_{MN}(t,x) = \begin{pmatrix} \rho^{-1/2}e^{2k}\eta_{\mu\nu} & 0 \\ 0 & \rho g_{\bar{m}\bar{n}} \end{pmatrix}, \tag{1}$$

where $M, N, \ldots \in \{t, x, y, z\}$; $\mu, \nu, \ldots \in \{t, x\}$; $\bar{m}, \bar{n}, \ldots \in \{y, z\}$; $\det g_{\bar{m}\bar{n}} = 1$; $\eta_{\mu\nu} = \mathrm{diag}\,(-1, +1)$ and k and ρ are some real functions. Note that $g = g^t$, which together with $\det g = 1$ means that g can be viewed as $SL(2, \mathbb{R})/SO(2)$-valued. The Einstein equations imply the wave equation $\eta^{\mu\nu}\partial_\mu\partial_\nu\rho = 0$ so that in the case of timelike $\partial_\mu\rho$ (i.e. for example in Gowdy's T^3-model [5]) the additional coordinate fixing $t = \rho$ (the non-stationary analog of Weyl's canonical coordinates) can be used to obtain a simplified version of the remaining Einstein equations:

$$\begin{aligned} \partial_t k = \frac{1}{8t}\mathrm{tr}({J_0}^2 + {J_1}^2) \quad , \qquad \partial_x k = \frac{1}{4t}\mathrm{tr}(J_0 \cdot J_1) \\ \partial_t J_0 - \partial_x J_1 \;\; = \;\; 0, \end{aligned} \tag{2}$$

where the currents $J_0,\ J_1$ are defined by

$$J_0 := tg^{-1}\partial_t g, \quad J_1 := tg^{-1}\partial_x g$$

and therefore obey the following integrability condition

$$\partial_t J_1 - \partial_x J_0 - \frac{1}{t}[J_1, J_0] - \frac{1}{t}J_1 = 0. \tag{3}$$

Eqs. (2) and (3) can be written as the compatibility conditions of a linear system [6] with a spacetime (ie. (t,x)-) dependent spectral parameter. It is essentially the $\frac{1}{t}J_1$-term in (3) which requires this spacetime dependence.

To obtain the Poisson brackets between the currents J_μ from the (relevant part of the) symmetry reduced Lagrangian

$$\mathcal{L} = Ct^{-1}\,\mathrm{tr}[J_0^2 - J_1^2], \tag{4}$$

where C is a constant, one has to take into account the symmetry $(g = g^t)$ and the unimodularity of g. One (and probably the most economical) way to do this, is to use the formalism described in [4]. An equivalent (less abstract

but more tedious) method is to parametrize g by two independent fields f and Φ:

$$g = \begin{pmatrix} f^{-1} & \Phi f^{-1} \\ \Phi f^{-1} & f + \Phi^2 f^{-1} \end{pmatrix}. \tag{5}$$

so that $\mathcal{L}$ becomes

$$\mathcal{L} = 2Ctf^{-2}[(\partial_t f)^2 + (\partial_t \Phi)^2 - (\partial_x f)^2 - (\partial_x \Phi)^2]. \tag{6}$$

Either way, the result is:

$$\{J_0(x) \overset{\otimes}{,} J_0(x')\} = \frac{1}{4C}[\Pi, \mathbf{1} \otimes J_0]\delta(x - x') \tag{7}$$

$$\begin{aligned}\{J_1(x) \overset{\otimes}{,} J_0(x')\} &= \frac{1}{4C}[\Pi, \mathbf{1} \otimes J_1]\delta(x - x') \\ &\quad + \frac{t}{4C}(2\Pi - \mathbf{1} \otimes \mathbf{1} + \varepsilon g \otimes g^{-1}\varepsilon)(x) \cdot \partial_x \delta(x - x')\end{aligned} \tag{8}$$

$$\{J_1(x) \overset{\otimes}{,} J_1(x')\} = 0, \tag{9}$$

where $\{A \overset{\otimes}{,} B\}_{\alpha\beta,\gamma\delta} \equiv \{A_{\alpha\beta}, B_{\gamma\delta}\}$ and Π denotes the permutation operator on $\mathbb{C}^2 \otimes \mathbb{C}^2$

$$\Pi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Leftrightarrow \Pi_{\alpha\beta,\gamma\delta} = \delta_{\alpha\delta}\delta_{\beta\gamma},$$

whereas the (2×2)-matrix ε is given by

$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{10}$$

Normally, non-ultralocal terms like those in (8) destroy the canonical formalism and lead to unresolvable ambiguities in the corresponding quantum theory. In the case at hand, however, these terms combine, roughly speaking, with the spacetime dependence of the spectral parameter of the linear system to yield unambigious results at the level of transition matrices [4]. This observation has been used to identify the conserved non-local charges that generate the Geroch group [7] with respect to the Poisson structure (7)-(9) [4] and to perform a consistent quantization in the case of cylindrical symmetry [8].

We now sketch how the above-mentioned generalized $SL(2, \mathbb{C})$-chiral model emerges within the Ashtekar formulation, thereby fixing our notation:

Indices from the middle (beginning) of the alphabet will denote spacetime (internal) indices:

$$\begin{aligned} M, N, \ldots \in \{t, x, y, z\}; \quad m, n, \ldots &\in \{x, y, z\}; \quad \bar{m}, \bar{n}, \ldots \in \{y, z\} \\ A, B, \ldots \in \{0, 1, 2, 3\}; \quad a, b, \ldots &\in \{1, 2, 3\}; \quad \bar{a}, \bar{b}, \ldots \in \{2, 3\}. \end{aligned}$$

The inverse densitized dreibein $\tilde{e}^m_a = e^m_a \det(e^b_n)$ and the components of the Ashtekar connection A_{ma} satisfy the fundamental Poisson brackets

$$\{\tilde{e}^m_a(x), A_{nb}(x')\} = -i\,\delta_{ab}\delta^m_n \delta^{(3)}(x-x')$$

and are subject to the first class constraints (with $a, b, \ldots$ raised with δ^{ab})

$$\begin{aligned} \mathcal{H} &:= \varepsilon^{abc} F^a_{mn} \tilde{e}^{mb} \tilde{e}^{nc} \approx 0 \\ \mathcal{C}_m &:= F^a_{mn} \tilde{e}^{na} \approx 0 \\ \mathcal{G}^a &:= D_m \tilde{e}^{ma} \approx 0 \end{aligned}$$

leading to the total Hamiltonian (without surface terms)

$$H_{\text{tot}}\,(\underset{\sim}{N}, N^m, \Lambda_a) = \int_\Sigma d^3x \left(\frac{1}{2} \underset{\sim}{N} \mathcal{H} + i N^m \mathcal{C}_m + \Lambda_a \mathcal{G}^a \right).$$

As usual, $\underset{\sim}{N} = N/\det(e^a_m)$, N^m, D_m and F_{mna} are the densitized lapse function, the shift vector, the covariant derivative with respect to A_{ma} and the corresponding field strength, respectively.

The reduction can now be divided into four main steps (see [2, 3, 9] for details):
(i)*A first gauge fixing:* We use adapted coordinates such that the two Killing vectors are given by the coordinate vector fields $\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z}$, implying the (y, z)-independence of all phase space variables. Subsequent imposition of the partial gauge fixing conditions $\tilde{e}^x_{\bar{a}} = \tilde{e}^{\bar{m}}_1 = 0$ breaks part of the $SO(3)$- and diffeomorphism invariance, while solving the resulting second class constraints requires $A_{\bar{m}1} = A_{x\bar{a}} = 0$ [3] and leaves us with a reduced phase space consisting of the canonical pairs

$$\begin{aligned} A := A_{x1} \quad &, \quad E := \tilde{e}^x_1 \\ A_{\bar{m}\bar{a}} \quad &, \quad \tilde{e}^{\bar{m}}_{\bar{a}} \end{aligned}$$

and the remaining first class constraints

$$\begin{aligned} G &:= \partial_x E + \varepsilon^{\bar{a}\bar{b}} A_{\bar{m}\bar{a}} \tilde{e}^{\bar{m}}_{\bar{b}} \approx 0 \\ C &:= A \partial_x E - \tilde{e}^{\bar{m}}_{\bar{a}} \partial_x A_{\bar{m}\bar{a}} \approx 0 \\ H &:= -2\varepsilon^{\bar{a}\bar{b}} F_{x\bar{m}\bar{a}} \tilde{e}^{\bar{m}}_{\bar{b}} E + F_{\bar{m}\bar{n}3} \tilde{e}^{\bar{m}}_{\bar{a}} \tilde{e}^{\bar{n}}_{\bar{b}} \varepsilon^{\bar{a}\bar{b}} \approx 0, \end{aligned}$$

where $\varepsilon^{\bar{a}\bar{b}} = -\varepsilon^{\bar{b}\bar{a}}$, $\quad \varepsilon^{23} = +1$.
(ii)*New variables:* Defining the (2×2)-matrices

$$\begin{aligned} B_0 &= (B_0)_{\bar{n}\bar{m}} := i\, K^{\bar{n}}_{\bar{m}} := i\, A_{\bar{m}\bar{a}} \tilde{e}^{\bar{n}}_{\bar{a}} \\ B_1 &= (B_1)_{\bar{n}\bar{m}} := J^{\bar{n}}_{\bar{m}} := \varepsilon^{\bar{a}\bar{b}} A_{\bar{m}\bar{a}} \tilde{e}^{\bar{n}}_{\bar{b}}, \end{aligned}$$

one finds the following Poisson brackets

$$\begin{aligned}
\{B_0(x) \overset{\otimes}{,} B_0(x')\} &= [\Pi, B_0 \otimes \mathbf{1}]\delta(x - x') \\
\{B_1(x) \overset{\otimes}{,} B_0(x')\} &= [\Pi, B_1 \otimes \mathbf{1}]\delta(x - x') \\
\{B_1(x) \overset{\otimes}{,} B_1(x')\} &= [\Pi, B_0 \otimes \mathbf{1}]\delta(x - x').
\end{aligned}$$

This together with $\{B_\mu, K\} = \{B_\mu, J\} = 0$, where $K := K^{\bar{m}}_{\bar{m}}$ and $J := J^{\bar{m}}_{\bar{m}}$, implies that the corresponding traceless (i.e. $sl(2,\mathbb{C})$-valued) parts

$$\begin{aligned}
A_0 &:= B_0 - \frac{1}{2}\mathrm{tr}B_0 \cdot \mathbf{1} = B_0 - \frac{1}{2} i\, K \cdot \mathbf{1} \\
A_1 &:= B_1 - \frac{1}{2}\mathrm{tr}B_1 \cdot \mathbf{1} = B_1 - \frac{1}{2} J \cdot \mathbf{1}.
\end{aligned}$$

satisfy the Poisson brackets

$$\begin{aligned}
\{A_0(x) \overset{\otimes}{,} A_0(x')\} &= [\Pi, A_0 \otimes \mathbf{1}]\delta(x - x') &(11)\\
\{A_1(x) \overset{\otimes}{,} A_0(x')\} &= [\Pi, A_1 \otimes \mathbf{1}]\delta(x - x') &(12)\\
\{A_1(x) \overset{\otimes}{,} A_1(x')\} &= [\Pi, A_0 \otimes \mathbf{1}]\delta(x - x'). &(13)
\end{aligned}$$

(iii) *The (nontrivial) equations of motion:* The time-dependence of A_μ follows from $\partial_t A_\mu = \{A_\mu, H_{tot}(\underset{\sim}{N})\}$ with[1] $H_{\rm tot}\,(\underset{\sim}{N}) = \int dx \left(\frac{1}{2}\underset{\sim}{N} H\right)$, yielding

$$\begin{aligned}
\partial_t A_0 &= \partial_x(\underset{\sim}{N} E A_1) &(14)\\
\partial_t A_1 &= \partial_x(\underset{\sim}{N} E A_0) - \underset{\sim}{N}[A_1, A_0]. &(15)
\end{aligned}$$

(iv) *Further gauge fixing:* In order to fix the coordinates t and x completely, we demand $E = t$, which requires $\underset{\sim}{N} = \frac{i}{EK} = \frac{i}{tK}$ for consistency, while the condition $K = i(=\text{const.})$ implies $\partial_x N^x = 0$ and fixes the x-coordinate up to transformations $x \to x + f(t)$ [2]. This freedom can then be used to absorb N^x by making the choice $f(t) = \int_{t_0}^t dt' N^x(t')$ [9]. Since A_μ commutes with E and K, these gauge fixings do not alter the Poisson brackets (11)-(13) (i.e. the latter coincide with the corresponding Dirac brackets), which, in contrast to (7)-(9), are completely ultralocal. Furthermore, the equations of motion (14)-(15) simplify to the system

$$\begin{aligned}
\partial_t A_0 - \partial_x A_1 &= 0 &(16)\\
\partial_t A_1 - \partial_x A_0 + \frac{1}{t}[A_1, A_0] &= 0. &(17)
\end{aligned}$$

Comparing these equations with the system (2)-(3) of the coset model, one encounters a surprising similarity. However, a term analogous to the $\frac{1}{t}J_1$-term in (3), which essentially caused the coordinate dependence of the spectral parameter of the linear system encoding (2)-(3), is absent. Indeed, a

[1] The other two constraints C and G, which normally would also appear in $H_{\rm tot}$ with Lagrange multipliers iN^x and Λ, need not be considered here, because A_μ commutes with G and an additional gauge fixing (see (iv)) will require $N^x = 0$.

generalized zero-curvature condition with *constant* spectral parameter can be used to construct certain non-local charges, which are not conserved, but commute with the reduced Hamiltonian [2].

We now come to the main part of this letter, in which we will translate the $SL(2,\mathbb{C})$-model into metric variables, thereby revealing its relation to the coset model. To accomplish this, we follow ref. [10] and parametrize the vierbein $E_M{}^A$ as follows:

$$E_M{}^A = \begin{pmatrix} N & N^a \\ 0 & e_m{}^a \end{pmatrix} \quad \Leftrightarrow \quad E_A{}^M = \begin{pmatrix} N^{-1} & -N^{-1}N^m \\ 0 & e_a{}^m \end{pmatrix},$$

with $N^m \equiv N^a e_a{}^m$, so that the components of the Ashtekar connection can be written as

$$A_{ma} = -\frac{1}{4}\varepsilon_{abc}\left(2\Omega_{dbc} - \Omega_{bcd}\right)e_m^d + ie_{mb}\Omega_{0(ab)},$$

where

$$\begin{aligned} \Omega_{AB}{}^C &:= 2E_{[A}{}^M E_{B]}{}^N \partial_M E_N{}^C = -\Omega_{BA}{}^C \\ \Omega_{ABC} &:= \Omega_{AB}{}^D \eta_{DC} \end{aligned}$$

are the coefficients of anholonomy and the (square-)brackets denote (anti-)symmetrization containing a factor 1/2. It is now rather straightforward to repeat steps (i) to (iv) at the level of vielbein- (and finally metric-) components:

Using adapted coordinates and imposing $\tilde{e}^x_{\bar{a}} = \tilde{e}^{\bar{m}}_1 = 0$ in step (i) leads to

$$\begin{aligned} A \equiv A_{x1} &= -\frac{1}{2}\varepsilon_{\bar{b}\bar{c}}e^{\bar{n}}_{\bar{b}}\partial_x e_{\bar{n}\bar{c}} + \frac{i}{N}(\partial_t e^1_x - \partial_x N^1) \\ A_{\bar{m}\bar{a}} &= -\varepsilon_{\bar{a}\bar{b}}e^x_1 e^{\bar{n}}_{(\bar{b}}\partial_x e_{\bar{n}\bar{c})} \cdot e^{\bar{c}}_{\bar{m}} + i\,\frac{1}{N}e_{\bar{m}\bar{b}}e^{\bar{n}}_{(\bar{a}}(\partial_t - N^x\partial_x)e_{\bar{n}\bar{b})} \\ A_{x\bar{a}} &= -\frac{i}{2N}e_{\bar{n}\bar{a}}\partial_x N^{\bar{n}} \\ A_{\bar{m}1} &= -\frac{i}{2N}e^x_1 e^{\bar{a}}_{\bar{m}}e_{\bar{n}\bar{a}}\partial_x N^{\bar{n}}. \end{aligned}$$

This shows that the conditions $A_{\bar{m}1} = A_{x\bar{a}} = 0$ are equivalent to $\partial_x N^{\bar{n}} = 0$. In the case of two-surface orthogonal Killing vector fields, one can even choose coordinates such that $N^{\bar{n}} \equiv 0$ so that the vierbein and therefore the metric become blockdiagonal (cf. eq. (1)).

As described in (ii), one now calculates the matrices B_μ and obtains for their traceless parts

$$A_0 = -\frac{e^1_x}{2N}\rho g^{-1}(\partial_t - N^x\partial_x)g - \frac{i}{2}\varepsilon\partial_x(\rho g) \tag{18}$$

$$A_1 = -\frac{1}{2}\rho g^{-1}\partial_x g - i\frac{e^1_x}{2N}\varepsilon(\partial_t - N^x\partial_x)(\rho g) \tag{19}$$

with $\rho g_{\bar{m}\bar{n}} = e_{\bar{m}}^{\bar{a}} e_{\bar{n}\bar{a}}$, $\rho = \det(e_{\bar{m}}^{\bar{a}})$ (cf. eq. (1)) and ε as in (10).

As has already been pointed out in [2], the final gauge fixings in (iv) are nothing but the choice of the (non-stationary) Weyl coordinates described at the beginning of this letter. Indeed, $E \equiv e_1^x \det(e_m^a) = \rho$, so that the requirement $t = E$ is equivalent to the choice $t = \rho$. On the other hand, $\underset{\sim}{N} = N/(e_x^1 \rho) = N/(e_x^1 t)$, so that the consistency condition $\underset{\sim}{N} = \frac{i}{tK}$ and the requirement $K = i$ imply $N = e_x^1$, which together with $N^x = 0$ means that the metric is of the same form as in (1) with $t^{-1/2} e^{2k} = (N)^2 = (e_x^1)^2$. Taking this into account, eqs. (18)-(19) simplify to

$$
\begin{aligned}
A_0 &= -\frac{1}{2} J_0 - \frac{i}{2} \varepsilon \partial_x (tg) &\quad (20)\\
A_1 &= -\frac{1}{2} J_1 - \frac{i}{2} \varepsilon \partial_t (tg), &\quad (21)
\end{aligned}
$$

which is our main result.

It remains to verify whether the system (16)-(17) with A_μ as in (20)-(21) is equivalent to the system (2)-(3) and whether the Lagrangian (4) really reproduces the ultralocal Poisson brackets (11)-(13) for combinations such as (20)-(21):

The equivalence of the equations of motion follows from the identities

$$
\begin{aligned}
\partial_t A_0 - \partial_x A_1 &= -\frac{1}{2} (\partial_t J_0 - \partial_x J_1) \\
\partial_t A_1 - \partial_x A_0 + \frac{1}{t} [A_1, A_0] &= -\frac{1}{2} \left(\partial_t J_1 - \partial_x J_0 - \frac{1}{t} [J_1, J_0] - \frac{1}{t} J_1 \right) \\
&\quad + \frac{i}{2} \varepsilon g \left(-\partial_t J_0 + \partial_x J_1 \right),
\end{aligned}
$$

which are valid for any real symmetric and unimodular (2×2)-matrix $g(t, x)$. As for the Poisson brackets, one uses again the method described in [4] or, alternatively, the parametrization (5) and the corresponding Lagrangian (6) to infer

$$
\begin{aligned}
\{A_0(x) \overset{\otimes}{,} A_0(x')\} &= \frac{1}{8C} [\Pi, A_0 \otimes \mathbf{1}] \delta(x - x') \\
\{A_1(x) \overset{\otimes}{,} A_0(x')\} &= \frac{1}{8C} [\Pi, A_1 \otimes \mathbf{1}] \delta(x - x') \\
\{A_1(x) \overset{\otimes}{,} A_1(x')\} &= \frac{1}{8C} [\Pi, A_0 \otimes \mathbf{1}] \delta(x - x').
\end{aligned}
$$

Thus, both formulations are based on the same Poisson structure; it is only due to the particular combination of the (2×2)-matrices in (20)-(21) that all potentially non-ultralocal contributions exactly cancel.

This leads to the following conclusions:
(i) The $SL(2, \mathbb{C})$-chiral model, grown out of the Ashtekar formulation and equipped with an ultralocal Poisson structure, and the metric-induced

$SL(2,\mathbb{R})/SO(2)$-σ-model with its non-ultralocal Poisson brackets are completely equivalent formulations of the two Killing field reduction of general relativity. The relation between these two chiral models is displayed by eqs. (20)-(21), which allow it now to translate the results of one approach into the language of the other. As a particularly interesting application one could now compare the non-local charges constructed in [4] with those given in [2].
(ii) The relation (20)-(21) between the currents J_μ and A_μ suggests an interesting link to ref. [11]. There it has been shown that by a very similar change of variables in the $O(3)$-chiral model it is possible to preserve the form of the equations of motion while rendering the corresponding Poisson brackets ultralocal. In our case, the transition $J_\mu \to A_\mu$ does not *completely* preserve the form of the equations of motion (the $\frac{1}{t}J_1$-term is eliminated), but also leads to ultralocal Poisson brackets.
(iii) While the coset model formulation has natural generalizations to other coset spaces that appear in more complicated models of dimensionally reduced (super-)gravity [4, 8], the $SL(2,\mathbb{C})$-formulation does not immediately suggest such generalizations, since its existence (like the construction in [11]) relies on some peculiarities of (2×2)-matrices.

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